1. Objective

This experiment measures the resultant force applied by a curved rectangular duct to the air flowing within the duct, and to determine if a control-volume analysis based on the linear momentum principle is valid. The other objective is to determine whether the tangential velocity of air flow in the bend can be modeled by a simple idealization.

2. Procedure

In this experiment the apparatus used involves a 90^o bent rectangular duct mounted below the contraction section of the air flow bench, from which air is blown along the duct and exhausted to the atmosphere. It has several pressure taps along the outer, inner, and radial walls of the duct, which are connected to pressure transducers. The pressure taps and their geometry are shown in Figure 1. A barometer and a thermometer are utilized to measure the atmospheric pressure and the temperature.

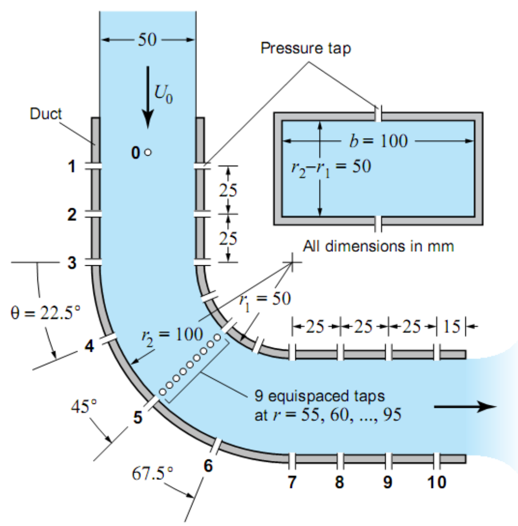


Fig. 1. Schematic of 90° bend in the rectangular duct.

Calibration of the pressure transducers is the first stage of the experiment procedures, and a monometer and a common airbox pressure are used at that moment. The atmospheric pressure recorded by the manometer was 99mb. Five calibration pressures are measured ranging from maximum flow to a lower flow of air.

Then the experiment data is taken by varying the flow and measuring pressures at all sides of the duct by changing which tubes are connected to the pressure meter. Three different flow rates are recorded by LABVIEW in order to measure the pressure distributions along the centerline of the duct and along the inner, outer, and radial walls of the duct wall. The upstream duct pressure tap 0 is connected to position 0 of the manifold and the airbox pressure tap is connected to position

15 on the transfer block. These two connections remain the same throughout the experiment. The calculation of the force of the duct is completed by the LABVIEW program. For the radial data, non-dimensional pressure coefficients are determined and tabulated in LABVIEW which formats all the data into a spreadsheet.

3. Theory

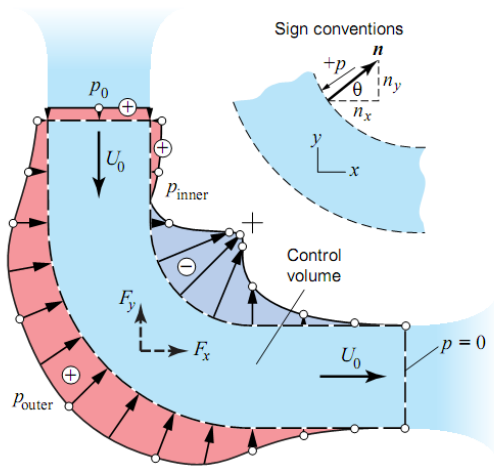


Fig. 2. Pressure distribution on the fluid in a 90° bend.

As shown on Fig. 2., it is adopted that the outer wall pressure will be positive because of the way in which the air flows around a bend and the inner wall pressure will be negative around the bend and. Based on the distances, it can be assumed that the air will flow faster around the inner wall of the duct compared to the outer wall. Also the net force, F, can be obtained using a control-volume analysis based on the principle of linear momentum. The following approximations are made: the flow is steady and two-dimensional, secondary flow effects and separation effects are neglected, and the flow enters and exits with a constant velocity. The pressure force components exerted by the duct on the air are:

$$F_x = 2(p_{box} - p_0) \times A \tag{1}$$

$$F_y = (2p_{box} - p_0) \times A \tag{2}$$

Note: the derivations for these equations can be found in the laboratory manual.

The free-vortex velocity distribution (C_p) and a Bernoulli equation at the entrance and any point along the radial plane, is given by:

$$C_p(r) = 1 - \left[\frac{r_2 - r_1}{ln\left(\frac{r_2}{r_1}\right)}\right]^2 \times \frac{1}{r^2} \tag{3}$$

Where r_1 = inner radius and r_2 = outer radius.

The inlet velocity is given by:

$$U_0 = \sqrt{\frac{2}{\rho} \times (p_{box} - p_0)} \quad (4)$$

Where ρ = air density.

4. Sample Calculations

Using flow rate #1 and tap#1:

Force Components

$$F_x = 2(900Pa - 119Pa) \times (0.05m \times 0.1m) = 7.81N$$

$$F_y = (2 \times 900Pa - 119Pa) \times (0.05m \times 0.1m) = 8.4N$$

Pressure Coefficient

$$C_p(r) = \frac{p(r) - p_0}{p_{box} - p_0} = \frac{-282\ Pa - 119Pa}{900Pa - 119Pa} = -0.5134$$

$$C_p(r) = 1 - \left[\frac{5203{,}42}{(55mm)^2}\right] = -0.7201$$

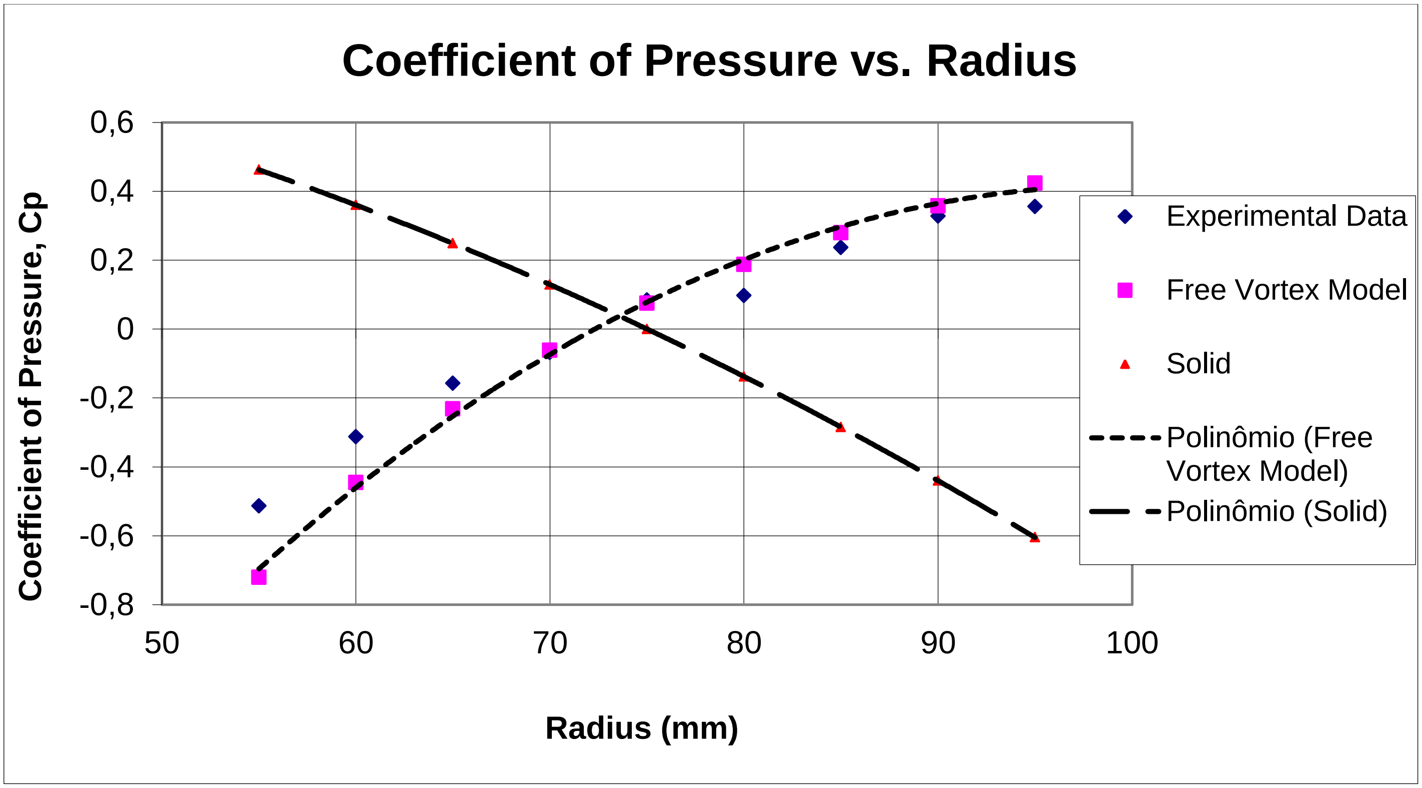
5. Results

Table 1. Experimental data (Flow rate # 1)			
Box pressure (mb):	9		
Upstream pressure (mb):	1,19		
Tap #	**Outer wall pressures (mb)**	**Inner wall pressures (mb)**	**Radial pressures (mb)**
1	1,04	1,26	-2,82
2	1,31	0,48	-1,25

3	2,14	-2,63	-0,04
4	3,36	-6,28	0,66
5	4,14	-7,71	1,85
6	4,09	-6,64	1,95
7	3,81	-3,78	3,04
8	2,13	0,15	3,75
9	0,63	0,61	3,97
10	0,21	0,41	

Forces found by integration (N)		**Forces found from control volume analysis (N)**	
Fx:	7.14	**Fx:**	7.81
Fy:	8.42	**Fy:**	8.4

Table 2. Coefficient of pressure data			
Rad Dist (mm)	**Cp (Exp)**	**Cp (Free Vortex)**	**Cp (Solid)**
55	-0,513444302	-0,720139654	0,462222222
60	-0,312419974	-0,445395126	0,36
65	-0,157490397	-0,231579279	0,248888889
70	-0,067861716	-0,061922949	0,128888889
75	0,084507042	0,07494712	0
80	0,09731114	0,186965242	-0,137777778
85	0,2368758	0,279803121	-0,284444444
90	0,327784891	0,357602166	-0,44
95	0,355953905	0,423443496	-0,604444444



6. Questions

2) Using standard atmospheric conditions (or actual recorded values of temperature and pressure), calculate the air density ρ and the inlet velocity U_0 based on your recorded pressure data.

$$U_0 = \sqrt{\frac{2}{\rho} \times (p_{box} - p_0)} = \sqrt{2(900Pa - 119Pa) \times \frac{1}{1.23\ \frac{kg}{m^3}}} = 35.63\ {}^{m}/_{s}$$

7) How do the models of air flow around the bend compare with each set of data as the flow rate decreases?

The two methods for calculating the forces differ more when the air flow rate increases. The flow becomes less like the free-vortex model as the flow rate increases.

7. Discussion

The forces calculated using the linear-momentum principle and integrating the pressure are differed. One reason of that difference could be the precision to calculate force by integrating over the arc length of the duct by integration method. However, the control volume analysis calculates the force at each point and takes into account the overall area of the duct rather than the arc length between particular points. Moreover, the control volume analysis takes into account a set of suppositions, which may be ideal but not valid in the real world.

The values of the Coefficient of Pressures vs. Radius graph showed that theoretical and experimental values of the pressure coefficients for the free vortex model calculated by LABVIEW program turned out to be very similar. A free-vortex model can be assumed for this experiment based on the graphical comparison of C_p and radius. The pressure coefficient C_p is proportional to radius because when C_p gets larger, the radius gets larger too.

8. Conclusion

The objective of the experiment was achieved and the resultant force on the air by the curved rectangular duct was computed for three different flows. The force was calculated using two ways of integration: integrating the outer and inner wall pressures over the surface length. The force was also calculated using a control-volume analysis in which was based on the initial and box pressure and area of the rectangular duct. The magnitude of the force was found to differ at high flow rates. From the coefficient of pressure vs. radius graph, it can be seen the free vortex model is better than the experimental data.